ECE 717
Exam 2 – Fall 2019

Name: ______________________________________

1. _____________

2. _____________

3. _____________

4. _____________

_____________ **(total score)**

1. True or False [18 points]. For each question, circle TRUE or FALSE. No explanation is required.

a)	If two realizations for the same transfer function have the same number of states (A is the same size for both), then there must exist a state transformation matrix T that transforms one realization into the other (and vice versa), even if the realizations are not minimal.	TRUE	FALSE
b)	If a linear time-invariant system is BIBO stable, then it is always Lyapunov stable.	TRUE	FALSE
c)	If a linear time-invariant system is asymptotically stable, then it is always exponentially stable as well.	TRUE	FALSE
d)	When using the input $u(t) = \sin(t)$, the output $y(t)$ of a system grows unbounded with time. We can conclude that the system is definitely NOT BIBO stable.	TRUE	FALSE
e)	When using the input $u(t) = \sin(t)$, the output $y(t)$ of the system is bounded. We can conclude that system is definitely BIBO stable.	TRUE	FALSE
f)	If $\dot{x}(t) = Ax(t)$ is asymptotically stable, then so is $x[k+1] = Ax[k]$ (same A matrix for both systems).	TRUE	FALSE

2. Minimal realizations [12 points].

a) Consider the following state-space realization:

$$\left[\begin{array}{ccccc|cc} 3 & 0 & 0 & 3 & 0 & 0 & 7 \\ 3 & 2 & 7 & 2 & 8 & 1 & 0 \\ 0 & 3 & 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Write down a minimal realization for this system.

b) Find a minimal realization for a system whose impulse response is $h(t) = 2e^{-t} - 3e^{-2t}$.

3. Lyapunov stability for a nonlinear system [10 points]. Consider the scalar nonlinear dynamical system:

$$\dot{x}(t) = -x(t) + \frac{1}{4}x(t)^3, \qquad x(0) = x_0$$

Prove that the point $\tilde{x} = 0$ is a locally stable equilibrium point.
Hint: use the Lyapunov function $V(x) = x^2$.

4. Evaluating quadratic integrals [10 points]. Suppose $\dot{x}(t) = Ax(t)$ and A is Hurwitz (all eigenvalues of A have a strictly negative real part). Suppose $Q = Q^\mathsf{T}$ is a symmetric matrix. We are interested in evaluating the integral

$$J(x_0) = \int_0^\infty x(t)^\mathsf{T} Q x(t)\,\mathrm{d}t$$

Where $x(t)$ is the state of the system at time t, assuming we start at $x(0) = x_0$. Prove that the value of this integral is given by:

$$J(x_0) = x_0^\mathsf{T} P x_0$$

where P satisfies the Lyapunov equation $A^\mathsf{T} P + PA + Q = 0$.