

ECE 717
Exam 1 – Fall 2019

Name: _____

(Each problem is worth 20 points)

1. _____

2. _____

3. _____

4. _____

_____ (total score)

1. Short answers. For each question, give a short answer. No proof or explanation is required.

a) If a square matrix is invertible, what can you say about its eigenvalues?

b) If a square matrix is diagonalizable, what can you say about its eigenvectors?

c) If a square matrix with real entries is symmetric, what can you say about its eigenvalues?

d) If a square and symmetric matrix is indefinite (neither positive definite nor negative definite), what can you say about its eigenvalues?

2. Transfer functions. Consider a continuous-time system with the transfer function:

$$Y(s) = \frac{1}{(s+1)(s+2)}U(s)$$

a) What is the impulse response of this system? Note: the Laplace transform of e^{-at} is $\frac{1}{s+a}$.

b) Find a state-space realization for this system

3. Matrix exponentials.

a) Compute the matrix exponential e^{At} , where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

b) Compute the matrix exponential e^{At} , where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

4. Controllability.

a) For which values of α is the following system controllable?

$$\dot{x}(t) = \begin{bmatrix} 1 & \alpha & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

b) We showed in class that if (A, B) is not controllable and its controllability matrix has rank $q < n$, we can find a state transformation matrix T such that $(A, B) \rightarrow \left(\begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ 0 & \hat{A}_{22} \end{bmatrix}, \begin{bmatrix} \hat{B}_1 \\ 0 \end{bmatrix} \right)$ where $\hat{A}_{11} \in \mathbb{R}^{q \times q}$. Prove that $(\hat{A}_{11}, \hat{B}_1)$ is controllable.