Agent-Level Optimal Cooperative Controllers for Dynamically Decoupled Systems with Output Feedback

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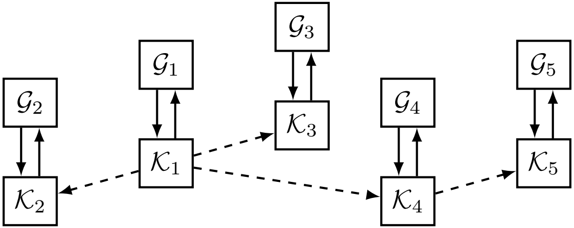
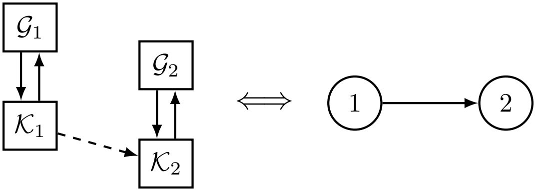
Problem setup

Find the globally optimal controller subject to communication constraints.



1. Subsystems (agents) are dynamically decoupled.
2. Controllers communicate along a known, directed, and *transitively closed* graph.

Two Body Example – Local Dynamics



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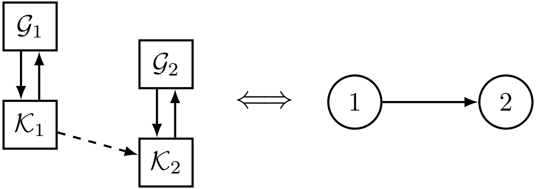
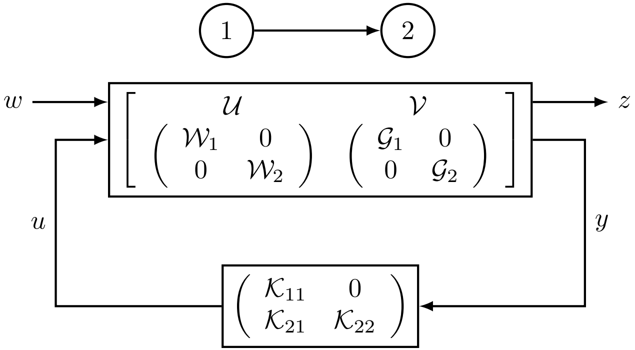


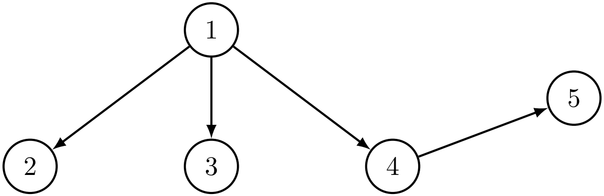
$$\mathcal{G}_1 = \left(\begin{array}{c|c} A_1 & B_{u_1} \\ \hline C_{y_1} & 0 \end{array} \right), \quad \mathcal{G}_2 = \left(\begin{array}{c|c} A_2 & B_{u_2} \\ \hline C_{y_2} & 0 \end{array} \right)$$

Two Body Example – Global Dynamics



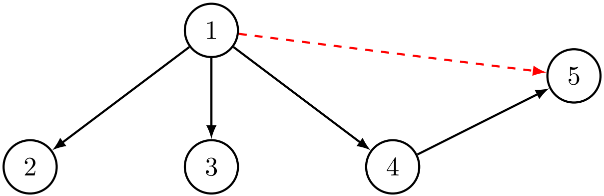
$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_1 & 0 \\ 0 & \mathcal{G}_2 \end{pmatrix} = \left(\begin{array}{cc|cc} A_1 & 0 & B_{u_1} & 0 \\ 0 & A_2 & 0 & B_{u_2} \\ \hline C_{y_1} & 0 & 0 & 0 \\ 0 & C_{y_2} & 0 & 0 \end{array}\right) \in \mathcal{S}; \quad \mathcal{K} \in \mathcal{S} = \begin{bmatrix} \times & 0 \\ \times & \times \end{bmatrix}.$$

Transitive Closure



$$\mathcal{S} = \begin{bmatrix} \times & 0 & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 \\ \times & 0 & \times & 0 & 0 \\ \times & 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times & \times \end{bmatrix}$$

Transitive Closure



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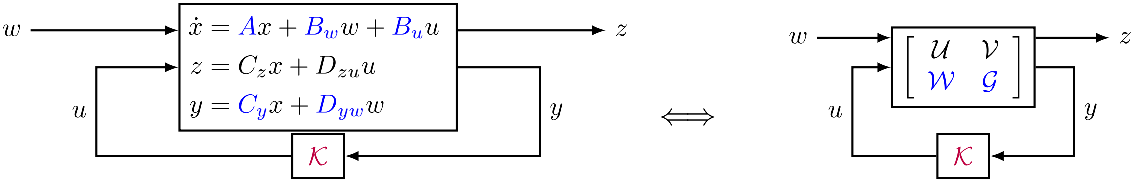
$\Rightarrow$ **Transitive Closure**

$$\begin{pmatrix} \mathcal{K}_{11} & 0 & 0 & 0 & 0 \\ \mathcal{K}_{21} & \mathcal{K}_{22} & 0 & 0 & 0 \\ \mathcal{K}_{31} & 0 & \mathcal{K}_{33} & 0 & 0 \\ \mathcal{K}_{41} & 0 & 0 & \mathcal{K}_{44} & 0 \\ \mathcal{K}_{51} & 0 & 0 & \mathcal{K}_{54} & \mathcal{K}_{55} \end{pmatrix} \in \mathcal{S}$$

Decentralized LQG Problem



$\mathcal{K} \Rightarrow$ Structure of $\mathcal{S}$ $\quad X \Rightarrow$ Block diagonal structure

$$\underset{\mathcal{K} \text{ stabilizes } \mathcal{G}}{\text{minimize}} \quad \left\| \mathcal{U} + \mathcal{V}\mathcal{K}(I - \mathcal{G}\mathcal{K})^{-1}\mathcal{W} \right\|_2^2$$
$$\text{subject to} \quad \mathcal{K} \in \mathbb{S}_{\mathcal{R}_\text{p}}.$$

We are solving a *constrained* Linear Quadratic Gaussian (LQG) problem.

Previous Work

1. Partial nestedness: In this setting, optimal controller is **linear**. [*Ho, Chu, 1972*]
2. Quadratic Invariance: In this setting, solving for the optimal linear controller is a **convex** problem. [*Rotkowitz, Lall, 2002*]

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Numerical Method [*Rotkowitz, Lall, 2006*]

Vectorize the problem and convert to a centralized problem.

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Vectorize the problem and convert to a centralized problem.

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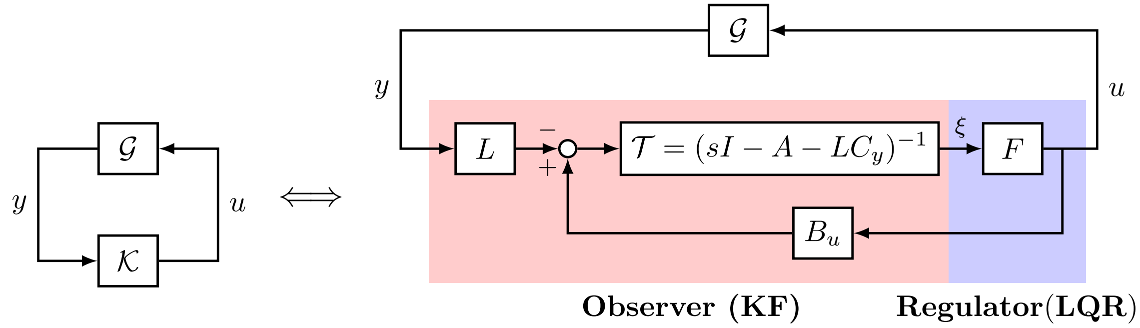
Separability approach [*Kim, Lall, 2015*]

Separate into smaller centralized LQG problems.

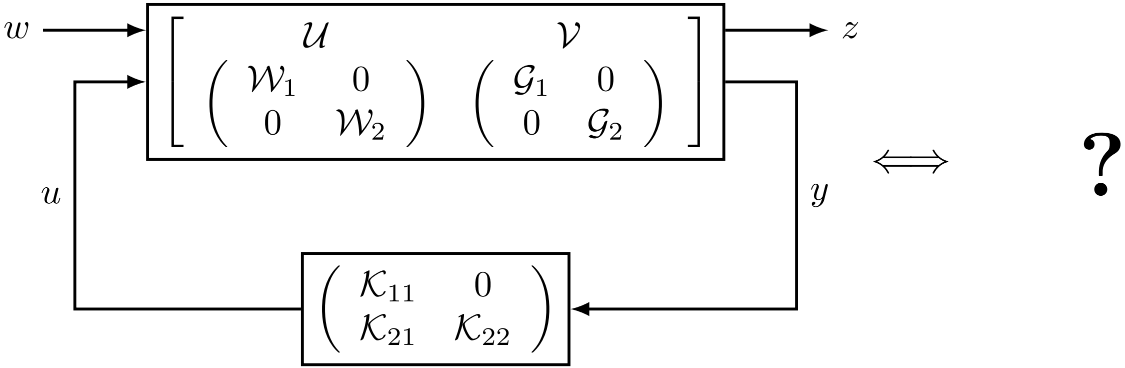
- $\mathcal{K}$ is minimal.

Centralized LQG



1. Familiar estimator-controller structure.
2. Optimal controller satisfies separation principle.
3. Provides intuition for non-LQG setting (EKF, UKF, etc.)

Decentralized LQG



Do we have a similar intuitive structure for decentralized system?

Our Goal

Obtain an **intuitive** representation for the **optimal** $\mathcal{K}$ that reveals the **minimal agent-level** structure.

Results

Global Architecture

1. Captures role of communication graph $\mathcal{S}$ and $\mathcal{S}^{-1}$.
2. Optimal controller satisfies modified separation principle.

Agent-Level Architecture

1. Explicit structure of controller, $\mathcal{K}_i$, of each agent.
2. Provides precise description of the signals transmitted between agents.

Results-I

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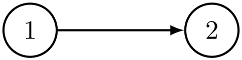
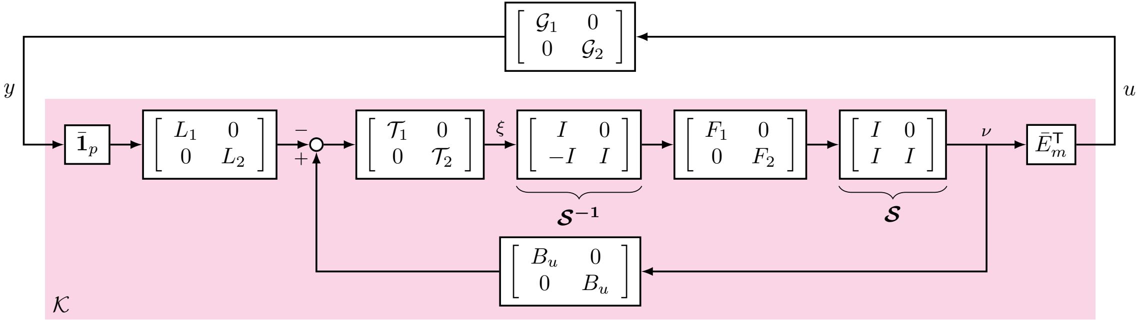
Global Controller $\mathcal{K}$ of Two Agents





$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \quad B_u = \begin{bmatrix} B_{u_1} & 0 \\ 0 & B_{u_2} \end{bmatrix}, \quad C_y = \begin{bmatrix} C_{y_1} & 0 \\ 0 & C_{y_2} \end{bmatrix}.$$

$$(X_1, F_1) := \mathrm{Ric}(A, B_u, C_z, D_{zu}), \tag{1a}$$

$$(X^{(2)}, F^{(2)}) := \mathrm{Ric}(A_2, B_{u_2}, C_{z_{22}}, D_{zu_{22}}), \tag{1b}$$

$$(Y^{(i)}, {L^{(i)}}^\mathsf{T}) := \mathrm{Ric}(A_i^\mathsf{T}, C_{y_i}^\mathsf{T}, B_{w_i}^\mathsf{T}, D_{yw_i}^\mathsf{T}), \quad \forall i \in \{1, 2\} \tag{1c}$$

$$\mathcal{T}_i = (sI - A - L_i C_y)^{-1}.$$

$$L_1 = \begin{bmatrix} L^{(1)} & 0 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} L^{(1)} & 0 \\ 0 & L^{(2)} \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 \\ 0 & F^{(2)} \end{bmatrix}.$$

Results-II

Global Architecture

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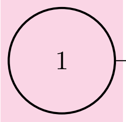
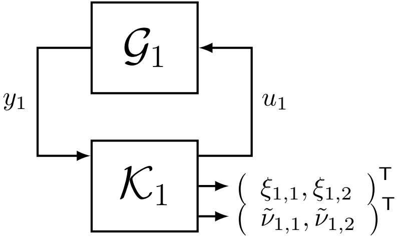
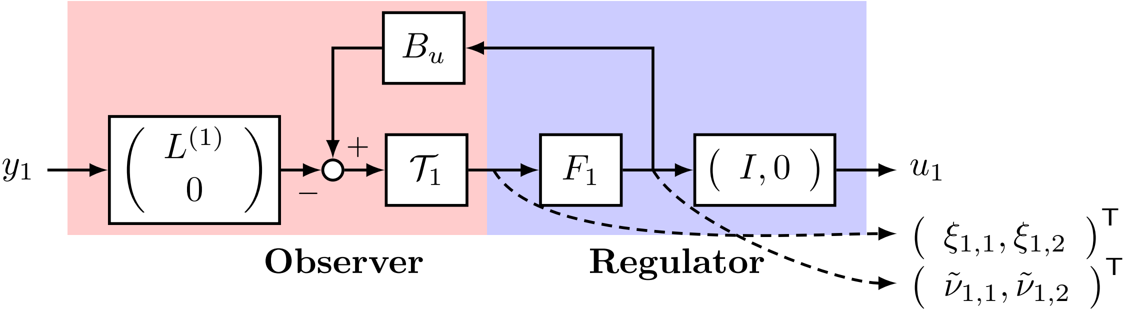
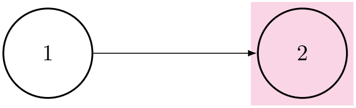
Local Controller of Agent 1

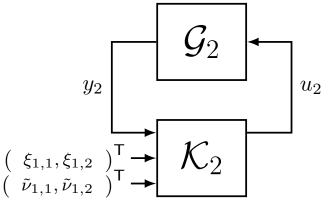


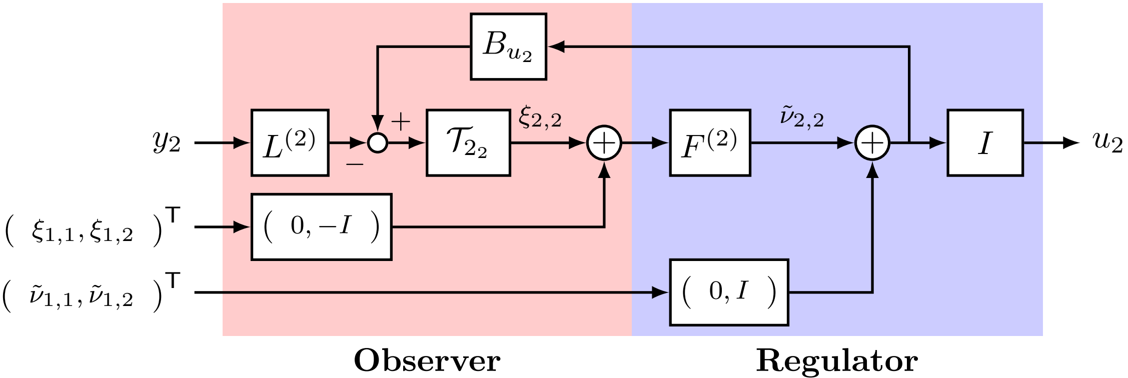




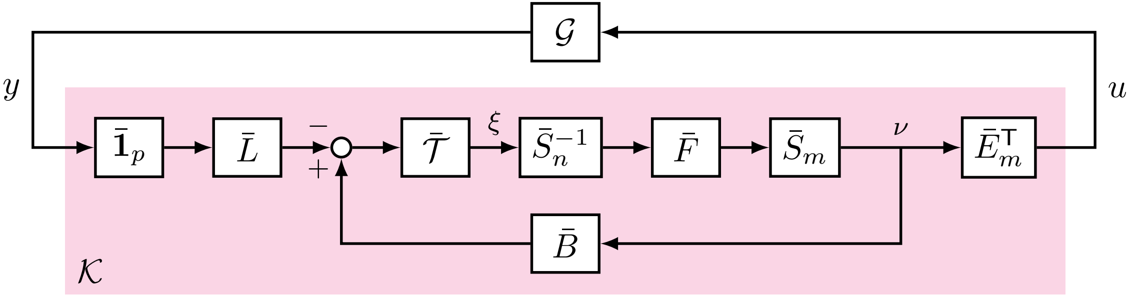
Local Controller of Agent 2





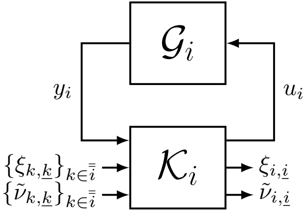


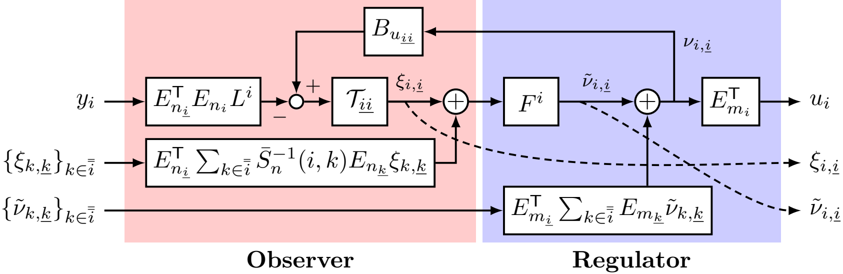
General Global Architecture



$$\bar{A} := I_N \otimes A, \qquad \bar{B} := I_N \otimes B_u, \qquad \bar{C} := I_N \otimes C_y,$$
$$\bar{S}_m := S \otimes I_m, \qquad \bar{S}_n := S \otimes I_n, \qquad \bar{S}_p := S \otimes I_p,$$
$$\bar{\mathbf{1}}_m := 1_N \otimes I_m, \qquad \bar{\mathbf{1}}_p := 1_N \otimes I_p.$$

General Local Architecture





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- The optimal controller's **global structure** ($\mathcal{K}$) resembles the optimal centralized LQG structure except it involves $\mathcal{S}$ and $\mathcal{S}^{-1}$.
- The individual agents' **local structure** ($\mathcal{K}_i$) has a simple minimal form and reveals which signals should be communicated along edges of the graph.

Summary

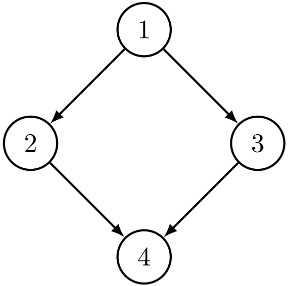
For the LQG problem associated with dynamically decoupled agents communicating on a known, directed, and transitively closed communication graph:

- The optimal controller's **global structure** $(\mathcal{K})$ resembles the optimal centralized LQG structure except it involves $\mathcal{S}$ and $\mathcal{S}^{-1}$.
- The individual agents' **local structure** $(\mathcal{K}_i)$ has a simple minimal form and reveals which signals should be communicated along edges of the graph.
- The optimal controller satisfies a separation principle.

Thanks!

Appendix-1a

Assumptions-Dynamically Decoupled System



$$\mathcal{S} = \begin{bmatrix} \times & 0 & 0 & 0 \\ \times & \times & 0 & 0 \\ \times & 0 & \times & 0 \\ \times & \times & \times & \times \end{bmatrix}$$

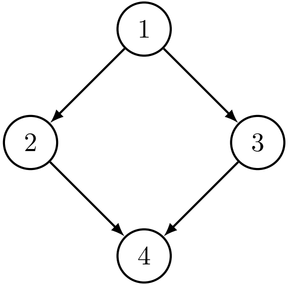
System Structure

The matrices A, B_u, C_y, B_w, D_{yw} are block diagonal and $D_{yu} = 0$.

1. $\implies A, B_u, C_y \in \mathcal{S}$ and $\left[\begin{array}{c|c} A & B_u \\ \hline C_y & 0 \end{array}\right] = C_y(sI - A)^{-1}B_u \in \mathcal{S}$.

Appendix-1b

Assumptions-Sparsity Pattern



$$\mathcal{S} = \begin{bmatrix} \times & 0 & 0 & 0 \\ \times & \times & 0 & 0 \\ \times & 0 & \times & 0 \\ \times & \times & \times & \times \end{bmatrix}$$

Controller Structure

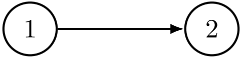
The controllers share information **instantaneously** along a directed acyclic graph (**DAG**) that is **transitively closed**.

1. $\implies \mathcal{S}$ is lower-triangular.
2. $\implies \mathcal{K}$ has sparsity pattern $\mathcal{S}$.

Appendix-2

Two Body Example

$$\{A_1, B_{u_1}, C_{y_1}, D_{yw_1}, B_{w_1}\}$$



$$\{A_2, B_{u_2}, C_{y_2}, D_{yw_2}, B_{w_2}\}$$

Noise Matrices-B_w and D_{yw}

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, B_w = \begin{bmatrix} B_{w_1} & 0 \\ 0 & B_{w_2} \end{bmatrix}, D_{yw} = \begin{bmatrix} D_{yw_1} & 0 \\ 0 & D_{yw_2} \end{bmatrix}.$$

Information Sharing Structure

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \mathcal{K}_{11} & 0 \\ \mathcal{K}_{21} & \mathcal{K}_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \mathcal{K} \in \mathcal{S} = \begin{bmatrix} \times & 0 \\ \times & \times \end{bmatrix}.$$

Appendix-3

Riccati Constants

$$(X^i, F^i) := \mathrm{Ric}(A_{\underline{ii}}, B_{u_{\underline{ii}}}, C_{z_{:\underline{i}}}, D_{zu_{:\underline{i}}}) \tag{2a}$$

$$(Y^i, {L^i}^\mathsf{T}) := \mathrm{Ric}(A_{\overline{ii}}^\mathsf{T}, C_{y_{\overline{ii}}}^\mathsf{T}, B_{w_{\overline{ii}}}^\mathsf{T}, D_{yw_{\overline{ii}}}^\mathsf{T}). \tag{2b}$$

For all $i \in [N]$, with F^i and ${L^i}^\mathsf{T}$, we also defined

$$L^{\bar{i}} := \mathrm{blkdiag}(\{L^j\}_{j \in \bar{i}})$$

$$\bar{L} := \mathrm{blkdiag}(\{L_i\}) \quad \text{where: } L_i := E_{n_{\bar{i}}} L^{\bar{i}} E_{p_{\bar{i}}}^\mathsf{T}$$

$$\bar{F} := \mathrm{blkdiag}(\{F_i\}) \quad \text{where: } F_i := E_{m_{\underline{i}}} F^i E_{n_{\underline{i}}}^\mathsf{T}.$$